

## Equations for Polymer Engineering

Rubber Elasticity:

$$\sigma = N_v kT (\lambda - \lambda^{-2})$$

$$\sigma = \frac{\rho RT}{M_c} \left( 1 - \left( \frac{2\bar{M}_c}{M_o} \right) \right) (\lambda - \lambda^{-2})$$

$$\sigma = \frac{\rho RT}{M_c} \phi^{1/3} (\lambda - \lambda^{-2})$$

$$\sigma_{0_x} = N_v kT (\lambda_x - \lambda_x^{-3} \lambda_y^{-2})$$

$$\sigma_{0_x} = \frac{\rho RT}{M_c} \phi^{1/3} \left[ \lambda_x - \frac{1}{\lambda_x^2} \right]$$

$$\sigma_{0_x} = \frac{\rho RT}{M_c} \left[ \lambda_x - \frac{1}{\lambda_x^2} \right]$$

$$\sigma = \left( \frac{f^* - 2}{f^*} \right) N_v RT (\lambda - \lambda^{-2})$$

$$\sigma = \left( \frac{f^* - 2}{f^*} \right) N_v RT \frac{r_i^2}{r_o^2} (\lambda - \lambda^{-2})$$

$$f = \left( \frac{\partial E}{\partial l} \right)_{T,v} - T \left( \frac{\partial S}{\partial l} \right)_{T,v}$$

$$f = \left( \frac{\partial E}{\partial l} \right)_{T,v} + T \left( \frac{\partial f}{\partial T} \right)_{l,v}$$

$$P = \frac{4d_0 N_v kT}{R_0 \lambda^3} * \left( \lambda - \frac{1}{\lambda^7} \right)$$

$$\Delta S = -\frac{N_c k}{2} [ (\lambda_x^2 + \lambda_y^2 + \lambda_z^2 - 3) + (\ln(\lambda_x \lambda_y \lambda_z)) ]$$

Thermodynamics of Mixing

$$\Delta H_m = V_m \left[ \left( \frac{\Delta E_1}{V_1} \right)^{1/2} + \left( \frac{\Delta E_2}{V_2} \right)^{1/2} \right]^2 (\nu_1 \nu_2)$$

$$\Delta H_m = N_1 \nu_2 \chi_{1,2} kT$$

$$\Delta G_{mix} = RT \left[ \ln(1 - \nu_2) + \left( 1 - \frac{1}{X} \right) \nu_2 + \chi_1 \nu_2^2 \right]$$

$$\chi_{1,2} = B_1 + \frac{V_1}{RT} (\delta_1 - \delta_2)^2$$

$$\chi_c = \frac{1}{2(1 - \nu_2)_c^2}$$

$$\chi_c = \frac{1}{2} \left( 1 + \frac{1}{X^{1/2}} \right)_c^2$$

$$\frac{\partial^2 \Delta G}{\partial^2 \mu_2^2} = \frac{1}{(1 - \nu_2)_c} + \left( 1 - \frac{1}{X} \right) + 2 \chi_c \nu_2$$

$$\pi = -\frac{RT}{V_1} \left[ \ln(1 - \nu_2) + \left( 1 - \frac{1}{X} \right) \nu_2 + \chi_1 \nu_2^2 \right]$$

$$x(t) = \frac{\rho_\infty - \rho_t}{\rho_\infty - \rho_a}$$

$$\left( \frac{1}{2} - \chi_c \right) = \Psi \left( 1 - \frac{\theta}{T} \right)$$

Crystallization & Kinetics

$$\frac{1}{T_f} - \frac{1}{T_f^o} = \left[ \frac{R}{\Delta H_f} \left( \frac{V_u}{V_1} \right) \nu_1 \right]$$

$$\Delta f_{Energy} = \left[ \Delta h_f \frac{(\Delta T)}{T_m^o} \right]$$

$$\Delta \Phi_v = 2bl\sigma + 2\nu_f ab\sigma_e$$

$$G_I = b_o a_o n_s i$$

$$G_I = G_o \exp(Q_D^*/RT) \exp(-K_{gI}^*/RT)$$

$$G_I = b(S_T g/aN)^{0.5}$$

$$S_K = \frac{1}{N_k} = \left(\frac{2g}{i}\right)^{0.5}$$

$$\frac{\Delta G_n}{kT_C} = \frac{C}{T_C} \left(\frac{T_m}{T_C}\right)^{n-1} \left(\frac{T_m}{\Delta T}\right)^{n-1}$$

$$\chi(t) = 1 - \exp(-kt^n)$$

Physical Aging Behavior/Viscoelasticity

$$\Delta \ln A_t = \frac{(B/f_o)(T-T_o)}{(f_o/\alpha_f) + (T-T_o)}$$

$$\frac{\ln \eta_1(t_1, T_1)}{\ln \eta_2(t_2, T_2)} = \frac{(B/f_o)(T-T_o)}{(f_o/\alpha_f) + (T-T_o)}$$

$$\frac{\ln \eta_1(t_1, T_1)}{\ln \eta_2(t_2, T_2)} = \frac{-8.86(T-T_o)}{101.6 + (T-T_o)}$$

$$\frac{\ln \eta_1(t_1, T_1)}{\ln \eta_2(t_2, T_2)} = \frac{-17.4(T-T_o)}{51.6 + (T-T_o)}$$

$$\eta = Ae^{\left(\frac{B V_o}{V_f}\right)}$$

$$f = f_o + \alpha_f (T-T_o)$$

General Thermodynamics

$$Q = \frac{m_1 - m_o}{m_o} \left(\frac{1}{\rho_s}\right)$$

$$\alpha = \frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_{P,I}$$

$$P = -\frac{1}{V} \left(\frac{\partial V}{\partial P}\right)_{I,T}$$

Chain Dimensions/Molecular Weight

$$\langle M_n \rangle = \frac{\sum N_i M_i}{\sum N_i}$$

$$\frac{Kc}{R_\theta} = \frac{1}{M_w} + \frac{1}{M_w} \frac{16}{3} \pi^2 \frac{\langle r^2 \rangle}{6\lambda^2} \sin^2 \frac{\theta}{2} + 2A_2c$$

$$p(r, n) = \left(2\pi \frac{n}{3} l^2\right)^{-3/2} 4\pi r^2 e^{\left(\frac{-3\langle r^2 \rangle}{2nl^2}\right)} dr$$

$$p(x, y, z, n) = \left(2\pi \frac{n}{3} l^2\right)^{-3/2} e^{\left(\frac{-3(x^2+y^2+z^2)}{2nl^2}\right)} dx dy dz$$

$$\langle r_g^2 \rangle = \frac{\langle r_o^2 \rangle}{6} = \frac{nl^2}{6}$$

$$\langle r^2 \rangle = nl^2 \frac{\langle 1 + \cos(180 - \tau) \rangle}{\langle 1 - \cos(180 - \tau) \rangle}$$

$$\langle r^2 \rangle = n\bar{l}^2 \left[ \frac{1 + \cos(180 - \tau)}{1 - \cos(180 - \tau)} \right] \left[ \frac{1 + \cos \langle \phi \rangle}{1 - \cos \langle \phi \rangle} \right]$$

$$\eta = KM_v^a$$

$$[\eta] = 2.5 \frac{4\pi}{3} \left(\frac{R_{eo}^2}{M}\right)^{3/2} M^{1/2} \alpha^3$$

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$$[\eta] = \Phi \left( \frac{\bar{r}^2}{M} \right)^{3/2} M^{1/2} \alpha^3$$

$$\frac{[\eta]}{[\eta_o]} = \alpha$$

$$\alpha^5 - \alpha^3 = 2C_m \Psi_1 \left( 1 - \frac{\theta}{T} \right) M^{1/2}$$

$$D_e = \frac{\lambda}{\tau}$$

$$\tau = \eta \gamma^n$$

$$\eta = \frac{\pi P R^4}{8 L Q}$$

$$\eta = A e^{E/RT}$$

$$\eta = A e^{V/V_0}$$

$$\frac{\ln \eta_1(t_1, T_1)}{\ln \eta_2(t_2, T_2)} = \frac{-17.4(T - T_o)}{51.6 + (T - T_o)}$$

$$\eta = A e^{\left( \frac{B V_o}{V_f} \right)}$$

$$f = f_o + \alpha_f (T - T_o)$$

$$\sigma = \sigma_\infty - \frac{K}{M}$$

$$G^* = \frac{\tau(t)}{\gamma(t)} = \frac{\tau_o e^{i\delta}}{\gamma_o}$$

$$\frac{G_f}{G_o} = 1 + 2.5\Phi + 14.1\Phi^2$$

$$\tau(t) = \tau_o \cos \omega t$$

$$w = 0.5 \sigma_{\max} \varepsilon_{\min}$$

### Viscoelasticity and Mechanics

$$E_{Young's} = 2 G_{Shear} (1 + \nu) = 3 B (1 - 2\nu)$$

$$\Delta \ln A_t = \frac{(B/f_o)(T - T_o)}{(f_o/\alpha_f) + (T - T_o)}$$

$$\frac{\ln \eta_1(t_1, T_1)}{\ln \eta_2(t_2, T_2)} = \frac{(B/f_o)(T - T_o)}{(f_o/\alpha_f) + (T - T_o)}$$

$$\frac{\ln \eta_1(t_1, T_1)}{\ln \eta_2(t_2, T_2)} = \frac{-8.86(T - T_o)}{101.6 + (T - T_o)}$$

### Fluid Dynamics (Rheology)

$$Q = \frac{\pi R^4 \Delta P}{8 \mu L}$$

$$Q = \left( \frac{Wh^2}{2(s+2)} \right) \left( \frac{h}{2m} * \frac{\Delta P}{L} \right)^s$$

$$Q_P = \frac{\pi Dh^3 \sin^2 \phi}{12 \mu} * \frac{\Delta P}{L}$$

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$$\dot{m} = \rho_{bulk} \pi D^2 \tan \phi \left( \frac{\pi}{4} \left[ D_B^2 - D_S^2 - \frac{E h}{\sin \phi} \right] \right)$$

$$Q_D = 1/2 \pi^2 D^2 N h \cos \phi \sin \phi$$

$$P = \frac{F^2}{9 \pi^2 r_1^2 r_2^2 \eta}$$

$$P = \frac{\eta 8 L Q}{\pi R^4}$$

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$$P = \frac{\eta 8 L Q}{\pi R^4}$$

$$F_0 = \frac{\alpha t}{x^2}$$

$$B_t = B_p^2$$

$$R_e = \frac{D V \rho}{\eta}$$

$$Q = V A$$

$$\eta = m(T) \dot{\gamma}^{(n-1)}$$

$$\eta = \mu(T)$$

$$\alpha = \frac{k}{\rho C_p}$$

$$t_{cooling} = \left( \frac{4 h^2}{\pi \alpha} \right) \ln \left[ \frac{8(T_m - T_w)}{\pi^2 (T_D - T_w)} \right]$$

Blow Molding

$$\frac{h_0}{h_f} = B_R D_R$$

$$D_R = \frac{v_f}{v_0}$$

$$B_R = \frac{R_f}{R_0}$$

$$X = \frac{Z}{R_0}$$

$$F = \frac{R_0 f_z}{\mu Q}$$

$$P = \frac{\pi R_0^3 \Delta p}{\mu Q}$$

Miscellaneous

$$E = E_f v_f + E_m v_m$$

$$\frac{1}{E} = \frac{v_f}{E_f} + \frac{v_m}{E_m}$$

$$\sigma_{yield} = 0.028 E_{Tensile}$$

$$E = 2(1 + \nu) G$$

$$E = 2(1 + 2\nu) B$$